Sample I

1.) Calculate the third roots of $z = 125(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4})$! (**15 points**)

2.) Give the polar form of $z = 1 + i$ and its tenth power! (**10 points**)

3.) Solve the linear system of equations $Ax = b$, and give the rank of the matrix A, where

$$A = \begin{bmatrix} 1 & 1 & 1 & -1 \\ 0 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

(**20 points**)

4.) Calculate the determinant of the following matrix!

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 9 \end{bmatrix}$$

(**5 points**)

5.) Give the definition of linear dependency and independency of a finite system of vectors! (**10 points**)

6.) Give the definition of vector space and subspace! (**10 points**)

7.) Execute the matrix operations $a^T A$ and $A^T a$, where

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & 4 \end{bmatrix}, \qquad a = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

(**10 points**)

8.) What will be x after the execution of the following loop? (**20 points**)

```
1 x = ones(1,6);
2 for i = 2:4
3       x(i) = 2*x(i-1);
4 end
```

Sample II

1.) Determine the inverse of the matrix A! (**15 points**)

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

2.) Solve the linear system of equations $Ax = b$ and calculate the rank of A, where

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 2 & 4 & 4 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

(**25 points**)

3.) Calculate the determinant of the following matrix!

$$\begin{bmatrix} -1 & 10 & 1 \\ 0 & -1 & -3 \\ -2 & 3 & 1 \end{bmatrix}$$

(**10 points**)

4.) Algebraic- and trigonometric form of complex numbers, conjugate of complex numbers. Theorem about the nth roots of complex numbers. (**20 points**)

5.) Determine the value of aa^T and a^Ta, where

$$a = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}.$$

(**10 points**)

6.) What will be the value of v after the execution of the following commands? (**20 points**)

```
1 v = ones(1,8);
2 for i = 2:5
3       v(i) = i*v(i-1)
4 end
```

Sample III

1.) Determine the eigenvalues and eigenvectors of the following matrix! (**15 points**)

$$\begin{bmatrix} 2 & 2 \\ -1 & 5 \end{bmatrix}$$

2.) Let $a = 2$, $t = 4$, $k_{(-)} = -3$, $k_{(+)} = 3$. Determine M_∞, ε_1 and ε_0! (**15 points**)

3.) Calculate the 1-norm and the ∞-norm of A! (**10 points**)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ -3 & 0 & 2 \\ 1 & -2 & 5 \end{bmatrix}$$

4.) Formulate the Lagrange interpolation problem, define the j.th basic Lagrangian polynomial and the Lagrangian polynomial. (**20 points**)

5.) Determine the parameters of the straight line $F(t) = a + bt$ which gives the best fit to the data in least squares sense (**20 points**):

t_i	1	2	2	3
f_i	2	4	$\frac{9}{2}$	6

6.) What will be the value of A and b after the execution of the code below? (**20 points**)

```
A = [1:4; 1:0.2:1.6; -ones(1,4)];
b = [10:-1:7];
for i = 1:3
    A(i,i)=i*(i+1)
    b(i)=b(i+1)+2
end
```

Sample IV

1.) Define the midpoint-, the trapezoidal- and Simpson's rule for an interval. (**15 point**)

2.) Define the Gauss normal equation (least square approximation), and define also the quantities in the equation.(The linear case for 12 points at most, and the general one for 15 points at most.) (**15 point**)

3.) Determine the minimal degree polynomial which fits to the following data. (**20 point**)

x_i	0	1
f_i	1	2
f_i'	3	3
f_i''	6	

4.) Let $a = 2,\ t = 4,\ k_{(-)} = -3,\ k_{(+)} = 3$. Determine the floating point form of 0.4 with truncation and rounding. (**10 point**)

5.) Calculate the 1-norm and ∞-norm of A. (**10 point**)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 3 \\ 1 & -11 & 5 \end{bmatrix}$$

6.) What will be the values of A and B after the execution of the underlying code? (**15 point**)

```
A = [1  1; -1  0; 1  2; 1  3];
B = A([1  3],:);
for i = 2:4
    A(i,:)=i*A(i,:);
end
```

7.) Determine the eigenvalues and eigenvectors of the following matrix. (**15 point**)

$$\begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$